Tailored Recommendations and Customized Pricing on a Matching Platform

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Abstract

Platforms not only mediate matches but work as information gatekeepers. When users who have private taste participate in a matching platform to find their partner, the platform asks them to provide matching-relevant information and, subsequently, aggregates and distributes the collected data back to each user to facilitate effective coordination of matches. How can a platform design information flow by which users form matches in a manner that is desirable to that platform? In this paper, I characterize a two-way communication approach that employs both verifiable and non-verifiable messages, and delineate the conditions under which a platform can (or cannot) achieve a socially optimal matching outcome using this communication protocol. In the platform that achieves such an outcome, users fully reveal their private taste, but the platform returns personalized and only filtered information back to each user in the form of a "Recommendation." I identify three key factors that enable such communication. I also demonstrate that a stable match can arise under certain conditions when the platform does not intervene in the communication between users, and users communicate with each other using verifiable messages. I then study the optimal customized pricing schedules of the platform. When the platform can fully customize its prices to each user, I show that full extraction of user surplus is possible if messages are verifiable and communications take place only through the platform. Lastly, as an application, I also study a two-way communication protocol with non-verifiable messages and demonstrate that communication strictly improves efficiency in any circumstances.

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1. Introduction

In recent decades, there has been a widespread increase in the availability of matching platforms.[1] On these platforms, two groups of users, such as job seekers and recruiters, participate in finding their matching partners from the other side. Although matching platforms are similar to traditional markets in that they both provide marketplaces in which two groups can meet, matching platforms exhibit a key innovative component: In addition to mediating matches, they operate as information gatekeepers. Platforms collect, aggregate, and distribute relevant information among users to facilitate effective coordination of matches.[2] When participating in a matching platform, users often have limited knowledge about their potential partners on the other side of the market and potential competitors on the same side; however, they know themselves better than anyone else. As an information gatekeeper, the platform asks users to provide their information when they register for the service and subsequently aggregates and distributes the collected data before users are engaged in their match decisions. Thus, the platform can indirectly influence the match outcome by controlling the information that users are asked to provide and to which they have access.

How can a platform design information flow to achieve its desired match outcome? If the preferences of users and the platform were perfectly aligned, the exchange of all decision-relevant information would be optimal for both the users and the platform. However, this is often not the case because of the scarcity of matching partners and the level of rivalry between the users. The platform takes all potential matches into consideration, while each user is only interested in his or her matching partner. For example, suppose there are two users on each side of the market: A and B on one side and C and D on the other side. If $((A, C), (B, D))$ is the desired matching outcome for the platform, the interest of the platform and some users are misaligned unless A prefers C to D and B prefers D to C. Owing to the presence of such misalignment in preferences, the interaction between user and platform incorporates a number of strategic aspects. First, users might not want to reveal their information to the platform. Second, a platform might want to provide only limited information to users. Finally, users might have an incentive to misuse the information they received from the platform and deviate from the behavior that the platform has intended to follow.

[1] Examples are Taskrabbit, Careerbuilder, and Airbnb.

[2] For example, at Freelancer.com, one of the largest online job-matching platforms, job seekers are required to upload their résumés and firms need to specify details about their job openings. After collecting and aggregating information from users, the platform presents recommended job opportunities to users.

To study the strategic aspects of a two-sided matching market, I consider a model that has the following features that mirror communications in matching platforms:

(i) There are finite numbers of users on both sides of the market. Each user has an attribute that is privately observed by the user. I characterize this attribute as a location in a unit interval. In the base model, users can costlessly provide their information to the platform in the form of a verifiable message.[3]

(ii) The platform can distribute the collected information back to users. I model this information flow to users as costless and non-verifiable messages such as "recommendations".[4]

(iii) Market is decentralized. Instead of being assigned to a partner, users have the freedom to choose their partners. Users receive higher payoffs if their partners are located closer to them.

The novelty of the current study is that it takes place in a setting in which users retain both information and authority. To incentivize users to fully reveal their information and to form the desired match, the platform needs to cautiously manage information flows back to the users.

I present two baseline models in the study. In the first model, I consider a benevolent platform that wants to achieve the socially optimal matching. Government-operated matching platforms can be good examples of this kind. In the model with a benevolent platform, the game proceeds as follows. After private information of the users is realized, users can choose how much of information to share with the platform. The platform collects and aggregates the information that is provided to it, and then provides information back to the users in the form of a "Recommendation". After communication between the users and the platform, a user on one side can make a proposition to a user on the other side. The user who receives a proposal can subsequently decide whether or not to accept this proposal. If users receive multiple proposals, they choose only one between the proposers or reject them all. If two users are matched, they equally share a match value that depends on their characteristics. Users who remain unmatched receive zero payoff as an outside option. Users care only about their match value, while the platform takes overall matches (or match values) into consideration to maximize

[3] Users cannot lie about their information. However, they can choose what information to disclose and what to withhold. An example of this type of message in our context is a résumé. I also study non-verifiable messages sent by users as an application of the base model.

[4] The message is delivered back to the users in the form of cheap talk. Messages do not directly affect utilities of players and there is no restriction on the set of messages a platform can send.

the social surplus, which is the sum of the match values. In the second model presented in section 5, I consider a profit maximizing platform that can fully customize its pricing schedule to each user after collecting verifiable messages from users. The timeline is the same as in the first model except that the platform offers customized pricing schedule instead of recommendations. Users must pay the amount specified in the schedule to match with a partner in the later model.

How much value does communication add to the platform? When can a benevolent platform mediate socially optimal matching through communication? What is the optimal customized pricing schedule that maximize profit of a greedy platform? The purpose of this study is to answer these questions and to better understand the value of communication and the strategic aspects arises in communication in a platform.

1.1. Main Results

Theorem 1 and Theorem 2 identify a sufficient condition and a necessary condition, respectively, under which a socially optimal matching is obtainable as an outcome of the recommendation game specified above. An equilibrium that achieves such an outcome has the following notable features.

(i) Users fully reveal their private information.

(ii) The platform provides personalized and filtered information back to each user.

The message sent from the platform to a user is personalized based on the information collected from the user, and it is filtered in so much as only limited and controlled information is delivered back to users. I show that a simple rule of recommendation can be used to achieve the desired outcome: The platform presents a recommended matching partner to each user. This recommendation, a message to a user, reveals only the identity of a partner that has the same rank as the user, while the exact location or rank is not revealed. Under the sufficient condition, this message rule of the platform results in two positive effects to users. First, users are willing to reveal their locations; otherwise, they would receive information that is less relevant to them. Second, users expect their recommended partners to be better matches for them than other users on the other side.

Theorem 1 and Theorem 2 state that the key components that directly influence whether such an equilibrium can be achieved are as follows: 1) The distance between the distributions of attributes of each side, 2) the uncertainty measure of each distribution, and 3) the population size. The three components work in the following way. The closer the distance between the distributions, the closer the distance between two locations of the same rank. On the other hand, if there is a high degree of uncertainty, the information provided by the platform becomes relatively more informative to users. Thus, according to the proximity of distributions and conditions of higher uncertainty, the recommendation generated by the platform is helpful to users, and they benefit from fully revealing their information and forming matches with recommended partners. Finally, a general condition can be obtained when the size of the population is large because, even if the distributions are close to one another, the expected distances between empirical distributions after observing one sample[5] on one side can still be large.

When the platform does not intervene in the users' information transmission and, instead, allows them to freely communicate with each other using verifiable messages, I show, in Theorem 3, that a stable matching can be attained as an outcome of a communication game under certain conditions. Under some regularity conditions related to the distributions, the condition that guarantees a stable matching can also be interpreted as a proximity condition of the distributions.

Theorem 4 describes an optimal pricing schedule when the platform can charge different prices for each pair of potential partners as a mean of to maximizing the profit. I derive a pricing schedule that consists of a function of collected messages collected from users, that fully extracts user surplus and supports an equilibrium with full revelation of private information. Full extraction of user surplus is particularly possible when messages are verifiable, and the platform customizes its charges to users and their potential partners. I observe that no information but price is sent back to users in the equilibrium in an equilibrium that achieves maximum profit under the optimal pricing schedule.

1.2. Implications for Information Design

The present research contributes to existing understanding in this domain by addressing the value of communication and information design problem in the context of a matching platform. If users believe their counterparties; i.e., their potential partners, share similar attributes to themselves overall, they will be more willing to fully disclose their information. This will allow the platform to achieve a socially optimal outcome using a simple communication protocol that discovers users' information and persuades them to form optimal matches. The study also addresses that price discrimination hurts welfare of platform users by allowing the platform to fully extract surplus. This also provides theoretical justification for the

[5]Each user's private observation of his or her location.

burgeoning use of personalized recommendations and customized pricing schedules on matching platforms. The results identify the key elements that will need to be taken into consideration for the platform to achieve its optimal outcome when it can adopt an interactive communication technology to system.

1.3. Related Literature

The current study lies at the intersection of two strands of literature: communication models with verifiable and non-verifiable messages and decentralized two-sided matching with a mediator.

Communication model with verifiable messages.– The core concept in this literature is "unraveling" of private information, originating from Grossman (1981) and Milgrom (1981). Upon receipt of a message, if the receiver infers the sender's type is the worst-case type among senders who could have sent this message, then the private information of the sender will "unravel," starting from the highest-type sender. This type of skepticism is generalized by Hagenbach et al. (2014). Hagenbach et al. (2014) studies conditions under which unraveling of private information is possible. My paper differs from Hagenbach et al. (2014) in various ways. First, while communication is public in Hagenbach et al. (2014), it is private in my model. Users report verifiable information to the platform in a private channel, and the platform sends only filtered information back to each user. The difference in communication channel and the existence of a strategic information gatekeeper results in players making different inferences after communication even if every user fully reveals his private information. Second, if there were no information intermediary, one could not apply skepticism in my model of horizontal information because no type of sender is superior to another. To apply skepticism, I need to design information transmission from the platform to generate orders among users' types. While most of the literature has focused on settings with vertical information with one-sided communication (Okuno-Fujiwara et al. (1990), Ben-Porath et al. (2017) and Quigley and Walther (2017)), Celik (2014) explores seller-buyer with horizontal attribute, and finds an equilibrium with partial unraveling of information. I provide an equilibrium with full disclosure of information.

Matching with intermediary.– The literature on two-sided matching with mediator has been quite influential but has focused primarily on cases in which 1) monetary transfers are possible or 2) mediators can restrict agents' actions. Damiano and Li (2007), Gomes and Pavan (2016) and Gomes and Pavan (2017) study optimal price discrimination of a monopolistic mediator. Kanoria and Saban (2017) and Arnosti et al. (2014) allow the platform to design the search environment so that it can restrict a user's ability to acquire information about users on the other side. The key difference with the current work is that these previous studies do not allow the platform to collect information from users. The platform chooses which side of the market users can search for their partners, assuming that search activity is decentralized and costly. The current study considers the case in which transfers are not possible, and the mediator does not have direct control over agents' actions. Instead, the platform can influence users' decisions by providing personalized choice-relevant information, and can elicit full information from users.

Two-sided matching with private information.– Hoppe et al. (2009) examines assortative matching under two-sided private information with costly signals; selection of costly signal occurs prior to matching, and costs create credibility in revelation. Poeschel (2013) considers a search-and-match model with private information and costless signal: agents who look for a match can exchange information via cheap talk messages. The study shows that truthful revelation is possible if players need to bargain over splitting match to be matched to each other. Messages become credible because higher types are expected to produce more, and therefore, pay more at the bargaining stage. In both studies, truthtelling behavior is driven by an (explicit or hidden) cost of signaling and an assumption about match value: a higher-type player produces a higher value. Hałaburda (2010) allows agents to commit to future relationship before the private information is observed, and provides an unraveling result under certain condition. In the model, truthful behavior can be interpreted in an informational context. Because the matching platform tailors its message in response to the agent's message, agents receive information that is not suitable for them if they provide false information.

Coarse-matching.– Chao and Wilson (1987), McAfee (2002) and Shao (2016) study a coarse-matching scheme in a two-sided matching problem with complete information and a continuum population. In coarse matching, a matchmaker sorts the population into finite classes, and the members are randomly matched within the classes. Hoppe et al. (2011) studies model of incomplete information with a continuum population, and shows that coarse matching can be implemented using monetary transfers among users and the matchmaker. As an application, I study an implementation problem of coarse matching without monetary transfer, but with communication. Using non-verifiable two-way communication, I show that coarse matching with two partitions can be implemented via communication in any environment.

Assortative matching.– Becker (1973) demonstrates that *positive assortative matching* (henceforth, PAM) is efficient if types are complementary. That is, match value is super-modular in attributes of the users who form a match. Shimer and Smith (2000) further investigates the conditions under which PAM is efficient when there is a search friction. My model builds on these papers by considering a horizontal location model with match values that are weakly concave and decreasing function of distance. Under this utility specification, PAM maximizes the sum of match values as long as all match values are non-negative.

1.4. Outline of the Paper

The rest of the paper is organized as follows. In the next section, I present the baseline model. Section 3 introduces a leading example. Section 4 analyzes the main theoretical results when the platform operates as a social planner. Section 5 studies the optimal pricing schedule of a profit-maximizing platform. Section 6 provides an extension of the model, and Section 7 concludes the paper. All proofs and supporting evidence are relegated to the appendices.

2. Model

I consider an economy that consists of workers and firms, each of which wants to match with one partner.[6] I use the term "user" to refer to a firm or a worker. The model deals with cases in which a matching platform cannot assign matches to users or use monetary transfers to incentivize users. Instead, a matching platform can collect information from, and distribute it to, users.

2.1. Primitives

Players.– There are n workers, n firms, and a platform. The set of workers (firms) is denoted by S_W^n (S_F^n, resp.). Worker i has a privately known area of specialty ω_i, which is independently distributed over the unit interval, according to a cumulative distribution function G. Firm j has a private taste θ_j, which is also independently distributed over $[0, 1]$, according to F. F and G are assumed to have positive densities f and g, respectively. I use the term "location" to refer to the type of a user. A user wants to match with another user on the other side of the market.

Utility.– Users are *horizontally* differentiated. They prefer to match with another user who has a similar taste to themselves. In terms of location, I assume that the closer the distance between users in a match, the higher the match value generated by the pairing. I denote the match value generated by pair (θ, ω) by $V(\theta, \omega)$, and assume each user expects to receive the generated match value if successfully matched to a partner. In the baseline model, I consider a platform that wants to

[6] The model can be used to study matching platforms targeting other types of users. I use the worker-firm terminology, although consumer-service provider or man-woman pairings could be equally relevant.

maximize the social surplus, the sum of all realized match values, generated in the platform. The case of a profit-maximizing platform is discussed at Section 6.

Condition 1. $V(\theta, \omega)$ is non-negative, continuous in θ and ω. It is concave and is a strictly decreasing function of the distance $|\theta - \omega|$.

The match value I use throughout the paper is $V(\theta, \omega) = 1 - (\theta - \omega)^2$. The main results of the paper holds with a slight modification in the statements for any V that satisfies Condition 1 and an assumption of a uniform convergence of utility functions. Details will be discussed in Theorem 1.

Communication.– I consider the following communication protocol that mirrors matching platforms in the real world. First, the platform requires users to send a verifiable message, à la Grossman (1981) and Milgrom (1981), to the platform. Examples of verifiable messages are résumés, employment history profiles, and job descriptions. After collecting information from the users, the platform aggregates the information and communicates with each user via a private channel. The messages from the platform to the users are not restricted, and are assumed to be non-verifiable. Examples of this type of message are "Recommendation for you" and "Instant match."[7] All messages are assumed to be costless in this model.

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1. The user sends a verifiable message to the matching platform.
2. After receiving messages from users, the platform provides a non-verifiable message to users.
3. After communication, workers apply to firms and firms accept or reject the application.

Table 1: Verifiable communication protocol

Assumption 1 (Verifiable message)**.** Type x user's set of messages, M_x, is a collection of all closed subsets of $[0, 1]$ with a restriction that $x \in m$, $\forall m \in M_x$.

In a setting in which verifiable messages are issued, users are allowed to withhold as much information as they like. However, they cannot lie about their actual locations in the sense that their type should be included in any message they send.

[7] Many online platforms provide "Instant match" options to users. Upon a user's request, the platform directly provides a match to another user on the other side of the market. Even if the platform matches users, it allows users to cancel the request after the match is provided. For example, in TaskRabbit, "Quick Assign" provides an instant match; however, the match provided can be canceled either if users do not engage in any further action regarding the match within two hours or if the assigned user cancels the match.

Timeline.– After firms and workers receive their private information, they can engage in two-way communication with the platform following the protocol presented above. After communicating with the platform, one side of the market, for example workers, can send a proposal to the other side of the market; in this case, a firm. The firm that subsequently receives the proposal can choose whether or not to accept the proposal. If the firm receives multiple proposals, it can accept at most one of them.

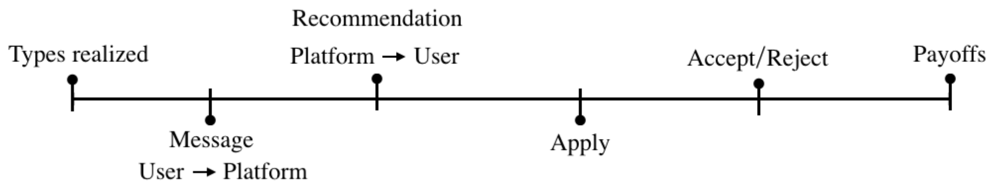

Figure 1: Timeline

The equilibrium concept is a perfect Bayesian equilibrium. To formalize the concept, denote the set of messages from the platform by M^p and let $M = \cup_{x\in[0,1]} M_x$, where M_x is the set of verifiable messages of user with a location x.

Worker's strategy.– Worker i's strategy is given by a pair of functions (ρ_i^c, ρ_i^s) where $\rho_i^c : [0,1] \to \Delta M$ with a restriction that $\mathbf{supp}(\rho_i^c(\omega)) \subset M_\omega,\ \forall \omega \in [0,1]$, and $\rho_i^s : [0,1] \times M \times M^p \to \Delta(S_F^n \cup \{\emptyset\})$. In words, worker i with ω chooses a verifiable message to send to the platform in the communication stage by ρ_i^c, and then chooses a firm to apply to in the selection stage. Workers are allowed to opt out, which is denoted by an action $\emptyset$.

Firm's strategy.– I denote firm j's strategy by a pair (σ_j^c, σ_j^s) where $\sigma_j^c : [0,1] \to \Delta M$ with a restriction that $\mathbf{supp}(\sigma_i^c(\theta)) \subset M_\theta,\ \forall \theta \in [0,1]$, and $\sigma_j^s : [0,1] \times M \times M^p \times A \to \Delta A$, where $A \subset S_W^n$ denotes the set of applications. In words, firm j with θ chooses messages from M_θ and send it to the platform in the communication stage. After the firm collects applications, it decides which worker to accept and which to reject. Again, choosing an $\emptyset$ has an interpretation of rejecting all applications and receiving an outside option.

Platform's strategy.– The platform's strategy is a collection of functions $\phi_i : M^n \times M^n \to M^P, \forall i \in S_W^n \cup S_F^n$. The platform decides which message to send to each user after collecting and aggregating messages from users.

I look for the following kind of equilibrium that is closely related both to the socially optima matching and to the profit-maximizing matching. In the equilibrium, all users fully reveal their type. The platform then generates a personalized and private recommendation that is based on an assortative matching, and all users follow the recommendations presented by the platform.

Definition 1 (Unraveling equilibrium)**.** An equilibrium is said to be an unraveling equilibrium if the strategies of users and the platform satisfy the following conditions:

(i) $\rho_i^c(x) = \{x\}$, $\forall i \in S_W^n$ and $\forall x \in [0,1]$;

(ii) $\sigma_j^c(x) = \{x\}$, $\forall i \in S_F^n$ and $\forall x \in [0,1]$;

(iii) $\phi_i(\boldsymbol{\omega}, \boldsymbol{\theta}) = j$ and $\phi_j(\boldsymbol{\omega}, \boldsymbol{\theta}) = i$, where ω_i and θ_j have the same rank in $\boldsymbol{\omega} = (\omega_1, \cdots, \omega_n)$ and $\boldsymbol{\theta} = (\theta_1, \cdots, \theta_n)$, respectively;

(iv) $\rho_i^s(\omega_i, \{\omega_i\}, j) = \{j\}$, $\forall i \in S_W^n$, $\forall \omega_i \in I_{[0,1]}$ and $\forall j \in S_F^n$; and

(v) $\sigma_j^s(\theta_j, \{\theta_j\}, \{i\}) = \{i\}$, $\forall j \in S_F^n$, $\forall \theta_j \in I_{[0,1]}$ and $\forall i \in S_W^n$.

The first and the second conditions state that all users fully reveal their information to the platform. Under the third condition, the platform recommends the lowest value firm to the worker with the lowest value, and vice versa. The platform also recommends the second lowest to the second lowest, and so on. A recommendation is said to be *assortative* if it satisfies (iii). Under (iv) and (v), each worker applies to his recommended firm and each firm only accepts the worker that the platform has recommended to it.

3. Example: Socially Optimal Matching through Communication

This section presents two examples. The first example demonstrates a construction of an unraveling equilibrium in a simplest setting possible, and the second example describes a setting in which an unraveling equilibrium does not exist.

Notations.– I first clarify some basic notation that will be used in the remainder of the paper. $\theta_{(k;n)}$ denotes the k^{th} order statistic[8] when there are n independent draws according to F. $F_{(k;n)}$ denotes its corresponding cumulative distribution function. $F_{(k;n)}$ is given by:

$$F_{(k;n)}(\theta) = \sum_{i=k}^{n} \binom{n}{i} F(\theta)^i (1 - F(\theta))^{n-i}$$

with density $f_{(k;n)} = n\binom{n-1}{k-1} F^{k-1}(\theta)(1 - F(\theta))^{n-k} f(\theta)$. On the worker side of the platform, $\omega_{(k;n)}$, $G(k;n)$, and $g_{(k;n)}$ are defined in a similar way.

[8] $\theta_{(1;n)} \leq \theta_{(2;n)} \leq \cdots \leq \theta_{(n;n)}$

Example 1: Heuristic derivation of an unraveling equilibrium

Consider a platform that consists of two firms and two workers whose locations are independently drawn from a uniform distribution. Suppose that user, k, on side G of the platform sends a verifiable message, m_k^G, to the platform, $G = F, W$ and $k = 1, 2$. I heuristically derive an unraveling equilibrium. Suppose that firm 1 in location θ_1 successfully convinces the platform that it is located at $\hat{\theta}$, and, subsequently, the platform recommends that firm 1 accepts worker 1, without loss of generality. From the assortative recommendation[9] of the platform in the equilibrium, firm 1 infers '$\omega_1 > \omega_2$' with the same probability that '$\theta_1 > \theta_2$', and '$\omega_1 < \omega_2$' with the same probability that '$\hat{\theta}_1 < \theta_2$'. That is, the location of ω_1, which is a random variable that is conditional on the platform's belief on the firm's location $\hat{\theta}_1$, can be expressed as a convex combination of two order statistics in the following way:

$$\hat{\theta}_1 \omega_{(2;2)} + (1 - \hat{\theta}_1)\omega_{(1;2)}.$$

At the same time, firm 1 also infers that the location of worker 2, the worker who is not recommended to the firm, is

$$\hat{\theta}_1 \omega_{(1;2)} + (1 - \hat{\theta}_1)\omega_{(2;2)}.$$

In cases in which only one application is presented to the firm, the firm accepts that applicant regardless of the identity of the worker. However, when the firm has two applicants, workers 1 and 2, it accepts worker 1 if, and only if,

$$\begin{aligned} &\hat{\theta}_1 \mathbb{E}[1 - (\theta_1 - \omega_{(2;2)})^2] + (1 - \hat{\theta}_1)\mathbb{E}[1 - (\theta_1 - \omega_{(1;2)})^2] \\ \geq\ &\hat{\theta}_1 \mathbb{E}[1 - (\theta_1 - \omega_{(1;2)})^2] + (1 - \hat{\theta}_1)\mathbb{E}[1 - (\theta_1 - \omega_{(2;2)})^2]. \end{aligned}$$

Similarly, worker 1, who receives a recommendation to apply to firm 1, updates his belief based on his report and the recommendation he received. When the platform believes his location is $\hat{\omega}$, he does not have any incentive to apply to firm 2 if

$$\begin{aligned} &\hat{\omega}_1 \mathbb{E}[1 - (\omega_1 - \theta_{(2;2)})^2] + (1 - \hat{\omega}_1)\mathbb{E}[1 - (\omega_1 - \theta_{(1;2)})^2] \\ \geq &\hat{\omega}_1 \mathbb{E}[1 - (\omega_1 - \theta_{(1;2)})^2] + (1 - \hat{\omega}_1)\mathbb{E}[1 - (\omega_1 - \theta_{(2;2)})^2]. \end{aligned}$$

[9] The social surplus is maximized when a firm with lower locations is matched with a worker with lower location.

Combining the above two conditions, all users follow their recommendations if

$$(1-2\hat{\theta})\mathbb{E}[(\theta_1-\omega_{(1;2)})^2] \leq (1-2\hat{\theta})\mathbb{E}[(\theta_1-\omega_{(2;2)})^2],\ \forall\theta_1 \in [0,1] \text{ and} \tag{1}$$

$$(1-2\hat{\omega})\mathbb{E}[(\theta_{(1;2)}-\omega_1)^2] \leq (1-2\hat{\omega})\mathbb{E}[(\theta_{(2;2)}-\omega_1)^2],\ \forall\omega_1 \in [0,1]. \tag{2}$$

Now, working backward in equilibrium, it is easy to verify that the above inequalities hold true when $\hat{\theta}=\theta_1$ and $\hat{\omega}=\omega_1$. This is, because any θ_1 that is smaller than $\frac{1}{2}$ is closer to $\omega_{(1;2)}$ than it is to $\omega_{(2;2)}$, and vice versa. Thus, the user who truthfully reported her location has no incentive to deviate from forming a match with the recommended partner. If all users truthfully reveal their location and follow the recommendations of the platform, the platform maximizes its utility by presenting an assortative recommendation. This is true for any individual utility that is a concave function of distance and the platform receives a positive match value from each match. I discuss this in more detail in the remark presented in Section 4.

To verify the incentive of users to fully reveal their location, recall the expected utility of a user, say a firm, when it successfully makes the platform believe its location is $\hat{\theta}$ while all other users fully reveal their locations. The expected utility is given by

$$\hat{\theta}_1\mathbb{E}[1-(\theta_1-\omega_{(2;2)})^2]+(1-\hat{\theta}_1)\mathbb{E}[1-(\theta_1-\omega_{(1;2)})^2].$$

Here, if θ_1 is located closer to $\omega_{(2;2)}$ than it is to $\omega_{(1;2)}$, the firm benefits from pretending to have higher $\hat{\theta}_1$, and vice versa. Thus, there is a threshold location $\bar{\theta}$ that a firm with location $\theta>\bar{\theta}$ will be strictly better off by pretending to have higher location and strictly worse off by doing the opposite. The incentive problem of a firm with location $\theta<\bar{\theta}$ is exactly the opposite. Thus, when a firm sends a message m to the platform, the platform rationally infers that the firm is located at

$$\theta' = \arg\min_{x\in m}|x-\bar{\theta}|.$$

As the platform skeptically believes the firm has a location that is closer to $\bar{\theta}$ and the message is verifiable, the firm can only successfully pretend to have a location of $\theta' \in [\theta_1, 1-\theta_1]$. (or $[1-\theta_1,\theta_1]$ if $\theta_1 \geq \frac{1}{2}$). However, the firm cannot benefit from this possible deception because this incurs a higher probability that it will receive a recommendation of a partner who is not suitable for the firm as stated above. The same reasoning can be applied to the other users, and I confirm that an unraveling equilibrium that achieves socially optimal matching exists in this case. As the next section will show, the key element that makes this unraveling possible is the distance between the two distributions of the workers and firms.

This is because users are more likely to believe that the recommended users share a similar location when the overall distributions are "close to" each other.

Example 2: Necessary condition

Just for an illustration purpose, let us relax the assumption that G and F are strictly increasing in this example, and consider the following two distributions[10]:

$$G(\omega) = \begin{cases} 2\omega & \text{if } \theta \leq \frac{1}{2} \\ 1 & \text{if } \theta \geq \frac{1}{2} \end{cases} \qquad F(\theta) = \begin{cases} 0 & \text{if } \theta \leq \frac{1}{2} \\ 2\theta - 1 & \text{if } \theta \geq \frac{1}{2} \end{cases}$$

First, consider the incentive of firm j with θ_j who has two applications. He follows

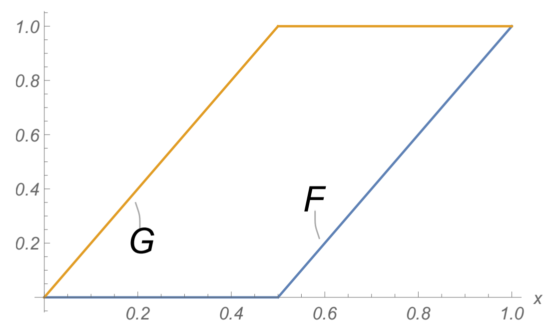

Figure 2: Example 1

recommendation of the platform if the condition (1) in the previous example holds. However, since the average distance to $\omega_{(2;2)}$ is closer than that to $\omega_{(1;2)}$ for any θ, firm j does not follow recommendation when he holds two applicants if and only if $\theta_j < F^{-1}(1/2) = 3/4$. Given that below median type firms are willing to accept a worker who is not recommended to them, consider worker i. Since the firm which is not recommended to him has $\theta_{(1;2)}$ with probability $G(\omega_i)$ and $\theta_{(2;2)}$ with probability $1 - G(\omega_i)$, i will be accepted by the firm which is not recommended to him with probability

$$G(\omega_i)F_{(1;2)}(F^{-1}(1/2)) + (1 - G(\omega_i))F_{(2;2)}(F^{-1}(1/2)) \equiv p(\omega_i).$$

Thus, worker i follows recommendation from the platform *if and only if*

$$\begin{aligned} &G(\omega_i)\mathbb{E}[1 - (\omega_i - \theta_{(2;2)})^2] + (1 - G(\omega_i))\mathbb{E}[1 - (\omega_i - \theta_{(1;2)})^2] \\ \geq &p(\omega_i)\left[G(\omega_i)\mathbb{E}[1 - (\omega_i - \theta_{(1;2)})^2] + (1 - G(\omega_i))\mathbb{E}[1 - (\omega_i - \theta_{(2;2)})^2]\right]. \end{aligned} \tag{4}$$

[10] I can easily find examples with the same result without the relaxation of assumption. For example, $F(\theta) = \theta$ and $G(\omega) = \omega^3$ also works well with this example but require more involved calculations.

The right hand side of the above inequality is weighted by $p(\omega_i)$ because his deviation is not successful with probability $1 - p(\omega_i)$ and receives zero from remaining unmatched. Since the left hand (the right hand side, resp.) side is strictly decreases (increases, resp.) in ω_i and the right hand side dominates[11] the left hand side at $\omega_i = \frac{1}{2}$, I see that $\exists \bar{\omega} < \frac{1}{2}$ such that worker with $\omega \geq \bar{\omega}$ strictly benefits from applying to the firm that is not recommended to him.

4. Socially Optimal Matching

The examples presented in the previous section highlight how the extent to which users are incentivized to abide by the platform largely depends on the proximity of the distributions. This section generalizes this idea and provides a sufficient condition and a necessary condition under which the platform can achieve a socially optimal outcome. Before begin, I specify the socially optimal matching. It can be achieved by a positive assortative matching: worker with higher attribute is matched with a firm with higher attribute. Given a realization $\theta^n = (\theta_1, \cdots, \theta_n)$, I denote the k^{th} smallest element of θ^n by $\hat{\theta}_{(k;n)}$. Similarly, $\hat{\omega}_{(k;n)}$ represents the k^{th} smallest element of $\omega^n = (\omega_1, \cdots, \omega_n)$.

Remark. An optimal matching is achieved when a worker with $\hat{\omega}_{(i;n)}$ is matched with a firm with $\hat{\theta}_{(i;n)}$, $\forall i \in \{1, \cdots, n\}$.

Proof. See **Appendix** for the proof. ■

4.1. Large Game

The first theorem predicts that the proximity requirement of the two distributions is, indeed, a sufficient condition to produce a socially optimal matching in large games through an unraveling equilibrium. Let Γ^n denote the game with n users on each side, $\sigma(F)$ refers to a standard deviation of random variable, the associated distribution function of which is F, and $\mathbf{Kolm}(F, G)$ denotes a Kolmogorov-Smirnov distance of F and G, which is given by $\mathbf{Kolm}(F, G) = sup_{x \in \mathbb{R}} |F(x) - G(x)|$.

Theorem 1 (Sufficiency)**.** $\exists N$ such that $\forall n \geq N$, a socially optimal matching can be obtained by communication in Γ^n if

$$\mathbf{Kolm}(F^{-1}, G^{-1}) < \min\{\sigma(F), \sigma(G)\}.$$

Proof. See **Appendix** for the proof. ■

[11] At $\omega_i = \frac{1}{2}$, $p(\omega_i) = \frac{3}{4}$, the value of the left hand side of (4) is $\frac{1}{2}$, and that of the right hand side is $\frac{23}{32}$.

I first note that the inverse of distributions is used to measure the distance. Suppose there is a continuum users on each side of the platform. In case of an assortative matching, a worker with ω is matched to a firm with $F^{-1}(G(\omega))$; hence, the user expects

$$1 - (\omega - F^{-1}(G(\omega)))^2. \tag{1}$$

Lemma 1 in **Appendix** states that the user's belief on the location of the recommended partner converges to a degenerated belief that assigns probability 1 to $F^{-1}(G(\omega))$ as n grows. Thus, I confirm that (1) is, indeed, an approximated the expected utility in large games. Now, denoting $G(\omega) = x$, I see that the distance between inverses is directly related to the expected utility in matching. Figure 3 shows the convergence of belief when $\omega = \frac{1}{2}$ with $F(\theta) = \theta^2$ and $G(\omega) = \omega$. Each graph depicts a CDF of the location of the recommended firm when a worker with location $\frac{1}{2}$ fully revealed his location to the platform in a situation in which there are there are n workers and n firms.

On the other hand, the standard deviation of F is related to the maximum

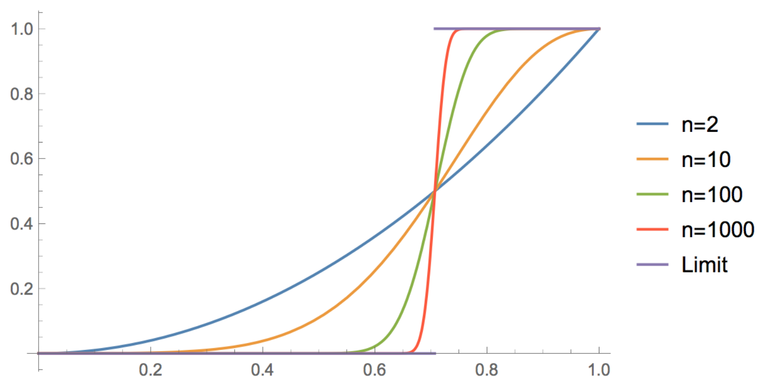

Figure 3: Convergence of belief

expected utility of the worker when he is accepted by a firm which is not recommended to him. If a worker with ω matches with a firm when he received no information from the platform, he expects an unconditional expected utility, $1 - \mathbb{E}_\theta[(\omega - \theta)^2]$. Since the platform provides only the identity of the recommended firm to the worker, the belief on the location of the firm that had not been recommended to the worker converges to the prior as n grows, and this results in the convergence of the expected payoff to the unconditional expected utility.

I also need to specify a belief system under which users cannot successfully masquerade as another type. I can construct such a belief system because the expected utility of a user with location x who successfully deceives the platform that his location is y exhibits increasing difference in (x, y) after the user receives a recommendation from the platform. As proof, I provide a way to implement a "skepticism" into this environment.

Finally, I observe that the number of users of the platform plays an important role. The platform cannot obtain an optimal outcome if there are not enough users even if the two distribution are close to each other. An extreme example of this kind is $F(x) = G(x) = x^3$ and $n = 2$. The reason for this is straightforward. First, in a small population, empirical distributions can still be far from each other, even if two distributions are adjacent to one another. Secondly, as the population size grows, more detailed matching becomes possible. That is, the uncertainty about the location of the recommended partner decreases.

To conclude the sufficiency part, I provide some examples which satisfy the sufficient condition. If both F and G are uniform distributions, $N = 1$ suffices. If $F(\theta) = \theta$ and $G(\omega) = \omega^2$, then $N = 3$ satisfies the conditions. However, $F(\theta) = \theta$ and $G(\omega) = \omega^3$ does not satisfy the condition.

I turn now to the conditions that are necessary for generating an unraveling equilibrium in large games. I first define $p \in [0, 1]$ in a way that

$$p = \mathbb{P}[x|(F^{-1}(x) - G^{-1}(x))^2 > \mathbb{E}_y[(F^{-1}(x) - G^{-1}(y))^2]],$$

where x and y independently follow the uniform distribution over the unit interval. Suppose a worker deviates from the recommendation and applies for a job at a firm that is not recommended to him. p denotes the measure of firms that are willing to accept the worker assuming there is a continuum firms. Thus, with a large population, p is the approximate probability that the worker will be successfully accepted by a firm in situations in which he has deviated from the recommendation.

Theorem 2 (Necessity)**.** If there exists N s.t. an unraveling equilibrium exists in Γ^n, $\forall n \geq N$, then $\nexists x^* \in [0, 1]$ s.t.

$$p > \frac{1 - (F^{-1}(x^*) - G^{-1}(x^*))^2}{1 - \mathbb{E}_y[(F^{-1}(y) - G^{-1}(x^*))^2]}.$$

Proof. See **Appendix** for the proof. ■

Theorem 2 predicts that the upper bound by the uncertainty measure in Theorem 1 can be relaxed to some extent. It is because feasible deviations in matching models should be bilateral deviations that form a blocking pair. Even if a worker has an incentive to apply to a firm that is not recommended to him, if p is low, he may not be accepted by that firm. However, it cannot be fully relaxed. If two distributions are far apart from each other, p tends to be large, and it becomes easier to find x^*, which satisfies the second condition of Theorem 2. To conclude this section, I provide an example that does not satisfy the necessary condition. If $F(\theta) = \theta^{\frac{1}{5}}$ and $G(\omega) = \omega^5$, then $x^* = \frac{2}{3}$ violates the necessary condition.

4.2. Limit Game: Assortative matching

The conditions provided in the previous section do not give a tight upper or lower bound on the distance of the distributions. This is mainly because the analysis takes an approximation approach to compare the expected utilities of users. Since the expected utilities of users do not behave in a consistent manner with changes in the number of population, predictions can only be obtained when there is a sufficiently large number of users in the population. However, in a game in which there is a continuum population, a more sharp prediction, which is a necessary and sufficient condition for an unraveling equilibrium, can be obtained.

Suppose now that there is a continuum users, with equal volume, on each side of the platform. As above, workers' (firms', resp.) locations are distributed according to a continuous and strictly increasing distribution G (F, resp.). As known in the literature of transportation theory, a socially optimal matching is achieved by the following PAM[12] and it is known to be unique if V is concave in $|\theta - \omega|$. Denote the set of workers' locations by Ω and the set of firms' locations by Θ.

Definition 2 (PAM)**.** A match $\mu : \Omega \to \Theta$ is positively assortative if $\mu(\omega) = F^{-1}(G(\omega))$

In PAM, users with the same quantile are matched to each other: worker a and firm b are partners to each other if and only if $G(\omega_a) = F(\theta_b)$. I denote the game by Γ^∞. The following proposition states a sufficient and necessary condition under which the platform achieves PAM through communication with the users.

Proposition 1. An unraveling equilibrium exist in Γ^∞ *if and only if* $\nexists x^* \in [0,1]$ s.t.

$$p > \frac{1 - (F^{-1}(x^*) - G^{-1}(x^*))^2}{1 - \mathbb{E}_y[(F^{-1}(y) - G^{-1}(x^*))^2]}.$$

Proof. See **Appendix** for the proof. ■

As predicted in the previous section, the following prediction also holds true.

[12] It is a solution of the Monge's problem. The proof appears in Chapter 3.2 of Rachev and Rüschendorf (1998) and Ekeland (2010).

Corollary 1. An unraveling equilibrium exist in Γ^∞ *if*

$$\mathbf{Kolm}(F^{-1}, G^{-1}) < \min\{\sigma(F), \sigma(G)\}.$$

Intuitively, a worker who fully revealed her location ω benefits from following the recommendation when her recommended firm, which is located at $F^{-1}(G(\omega))$ in PAM, is close to herself. With verifiable messages, the platform also can make users fully reveal their locations by applying skepticism as masquerading incentives of users are straightforward. For example, suppose that F is a standard normal distribution and G is a uniform distribution as shown in Figure 4 below. A worker of location ω has incentives to masquerade as $\omega' < \omega$ if $G(\omega) > F(\omega)$ and she has the exact opposite incentive if $G(\omega) < F(\omega)$. It is because she knows her recommended partner is below her location if $G(\omega) < F(\omega)$, and vice versa. Thus, whenever the two distributions cross each other, users have reversed direction of incentives. Taking this into consideration, the platform can rationally infer that a user is located at the worst case location possible among the claimed locations.

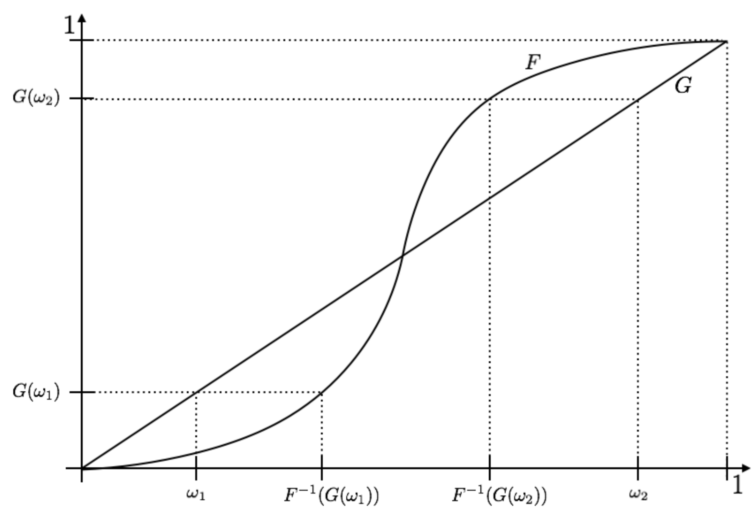

Figure 4: Masquerading incentives of users

Proposition 1 states that the necessary condition in the previous section is indeed a necessary and sufficient condition in a limit game with continuum users. However, this does not imply that the condition

$$p < \frac{1 - (F^{-1}(x^*) - G^{-1}(x^*))^2}{1 - \mathbb{E}_y[(F^{-1}(y) - G^{-1}(x^*))^2]}$$

can be sufficient for the existence of an unraveling equilibrium in large games. The probability that a deviating worker is accepted by a firm that is not recommended to him converges to p. However, since the rate of convergence is not known, so does the existence of a population threshold N.

4.3. Limit Game: Achieving an Ex-post Stable Matching

This subsection studies the cases in which the platform cannot influence the formation of matches by providing recommendations to users. Consider a laissez-faire economy in which the platform does not play a mediating role. Two groups of users, workers and firms, of the same volume are located in a unit interval according to G and F. Before studying an outcome of this game under incomplete information, consider first a complete information game as a benchmark case. I impose a regularity condition on F and G for the sake of simplicity of the matching outcome.

Assumption 2 (MLRP)**.** f and g bear monotone likelihood ratio property. That is,

$$\frac{f(x)}{g(x)} \text{ is non-decreasing in } x.$$

As also noticed in Clark (2007) and Flanders (2013), there is a unique stable matching of the following kind.

Remark (Stable Matching)**.** In the case of stable matching with perfect information, $\min\{f(x), g(x)\}$ measure of workers match to the firms with location x and remaining workers of location ω match to the firm with location $F^{-1}(1 - G(\omega))$.

Proof. See Proposition 3 of Flanders (2013) for further evidence. ■

Among the class of stable matches, I focus on matching that is fair in the sense that workers of the same location face the same lottery over firms. In a matching that is stable and fair, a worker with ω matches with a firm of the same location with probability $\min\{g(\omega), f(x)\}$ and matches with a firm of location $F^{-1}(1 - G(\omega))$ with probability $\max\{g(\omega), f(\omega)\} - \min\{g(\omega), f(\omega)\}$.

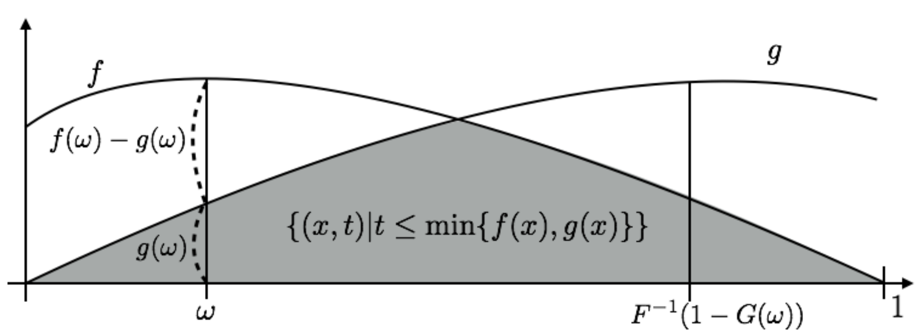

Figure 5: Stable matching

Now, suppose that locations represent the private information of each user. Before choosing their partners, users can engage in a public communication that discloses their locations using verifiable messages and then observe each other's disclosed locations. To examine a sufficient condition that achieves stable matching without mediation of the platform, I further impose a condition that highlights the main driving force of the sufficient condition.

Assumption 3 (Mirror)**.** f and g mirror each other across the midpoint, $\frac{1}{2}$. That is,

$$f(x) = g(1 - x).$$

A general sufficient condition without assuming this regularity condition will be presented and discussed in the Appendix.

Theorem 3. Stable matching is attainable if

$$\max\{f(x), g(x)\} \leq 2\min\{f(x), g(x)\}, \ \forall x \in [0, 1].$$

Proof. See **Appendix** for the proof. ■

The condition presented in Theorem 3 can be interpreted in a similar way to the condition stated in Theorem 1: The proximity of the two distributions guarantees the existence of fully revealing equilibrium. The reason for this is simple and intuitive. If all users fully reveal their locations, a worker of location ω expects to be matched to a firm with the same location with probability $\min\{g(\omega), f(\omega)\}$ and to a firm of location $F^{-1}(G(1-\omega))$ with probability $\max\{g(\omega), f(\omega)\} - \min\{g(\omega), f(\omega)\}$. If the two distributions are close to each other, the former probability and its resulting utility dominates the later, and thus, the user benefits from fully revealing her location.

5. Profit Maximizing Matching

Suppose now that the platform can customize its pricing schedule to each user based on the messages collected. The timeline is as follows: After collecting verifiable messages from the users, the platform provides a customized pricing schedule to each user. Observing the schedule, users then decide their matching partners. Here, a pricing schedule to worker i given a message profile $\boldsymbol{m}$ is a map $T_i^W(\boldsymbol{m}) : S_F^n \to \mathbb{R}$. Similarly, $T_j^F(\boldsymbol{m}) : S_W^n \to \mathbb{R}$ is defined as a pricing schedule to firm j given a message profile m collected from the both side of users. When worker i and firm j are matched at the price of $T_{i,j}$ to i and $T_{j,i}$ to j, each user respectively expects $1 - (\theta_j - \omega_i)^2 - T_{i,j}$ and $1 - (\theta_j - \omega_i)^2 - T_{j,i}$.

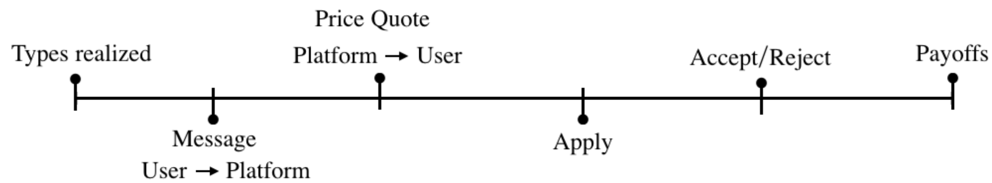

Figure 6: Timeline

How can the platform maximize its profit, the sum of prices from the matching pairs? Is it possible to achieve the first best outcome? If so, under what conditions is it possible? I begin the analysis with a simple pricing problem of a platform under location uncertainty and describe how the platform makes inferences about the unobserved locations based on their messages.

5.1. Example

Suppose there is one firm and one worker using a platform. If the locations θ_1 and ω_1 were perfectly observable by the platform, it can fully extract match surplus by charging $1-(\theta_1-\omega_1)^2$ to the worker and the firm. In this example, I show that the platform can achieve the same profit using verifiable messages even when locations are private information. Firstly, consider a pricing schedule by which the platform charges $1-(\hat{\theta}-\hat{\omega})^2$ to the users if it believes they are located at $\hat{\theta}$ and $\hat{\omega}$. Denote the message from the worker (the firm, resp.) by m_w (m_f, resp.).[13]. After receiving the message, the platform makes an inference that the location of the worker is $\hat{\omega}_1$, where

$$\hat{\omega}_1 = \arg\min_{\omega \in m_w} |\omega - \mathbb{E}_\theta \theta|$$

and the location of the firm to be $\hat{\theta}_1$ where

$$\hat{\theta}_1 = \arg\min_{\theta \in m_f} |\theta - \mathbb{E}_\omega \omega|.$$

That is, the platform skeptically makes an inference that the users are located at the points that are the closest to medians. To see how this skepticism works in equilibrium, consider the worker's problem when the firm fully revealed its location to the platform. If she successfully makes the platform believe that her location is $\bar{\omega}$, her expected payoff from this successful deception is

$$Q^W(\omega_1, \bar{\omega}) = \mathbb{E}_\theta[1-(\theta-\omega_1)^2 - (1-(\theta-\bar{\omega})^2)],$$

[13]Recall that m_i is constrained to be a closed subset of $[0,1]$ that contains ω_i.

where θ is located according to F. Here, $Q^W(\omega_1, \bar{\omega})$ decreases in $\bar{\omega}$ if $\bar{\omega} < \mathbb{E}_\theta \theta$ and increases if $\bar{\omega} > \mathbb{E}_\theta \theta$. That is, a worker in location ω_1 benefits from pretending to be located further away from $\mathbb{E}_\theta \theta$. Thus, the platform rationally infers that the true location of the worker is the element of m_w that is the closest to $\mathbb{E}_\theta \theta$ because the worker would not send such a message had the true location been closer to the endpoints.

5.2. Generalization: Full Extraction of User Surplus

The main reason why skepticism works in the above example can be attributed to the fact that the payoff from a successful deception, $Q^W(\omega, \bar{\omega})$, exhibits increasing difference: If ω benefits from mimicking $\bar{\omega}$, $\bar{\omega}$ cannot gain from mimicking ω. As shown in the proof of Theorem 1, this property allows the platform to figure out the true locations of users by inferring the worst-case location possible that would have sent the message. To generalize the idea, consider a platform with n workers and n firms. First, notice that if the platform observes the true locations of the users, it maximizes its profit when users form assortative matching. I can define users' payoffs from a successful deception, Q_n^F and Q_n^W, by

$$
\begin{aligned}
Q_n^F(\theta, \theta') =& \mathbb{E}[(\theta' - W_n(\theta'))^2 - (\theta - W_n(\theta'))^2] \text{ and} \\
Q_n^W(\omega, \omega') =& \mathbb{E}[(\omega' - \Theta_n(\omega'))^2 - (\omega - \Theta_n(\omega'))^2],
\end{aligned}
$$

where $W_n(\theta)$ is the expected location of the partner in an assortative matching when the platform believes the firm's location is θ and $\Theta_n(\omega)$ is defined in a similar way. In other words, a worker with ω expects to gain $1 - (\omega - \Theta_n(\omega'))^2$ and pay $1 - (\omega' - \Theta_n(\omega'))^2$ by successfully pretending to have a location of ω'.

Definition 3 (Worst-case location, Hagenbach et al. (2014))**.** A location, t, is said to be the worst-case location if $t \in wcl(m^G | Q_n^G) = \{t' \in m^G | \nexists t'' \in m^G,\ Q_n^G(t'', t') > Q_n^G(t'', t'')\},\ G = W, F$

Now, I state an optimal pricing schedule of the platform using the worst-case locations to infer true locations. Denote profiles of locations inferred by wcl operator by $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\omega}}$. That is, $\hat{\theta}_j \in wcl(m_j^F | Q_n^F)$ and $\hat{\omega}_j \in wcl(m_j^W | Q_n^W)$ for $j \in \{1, \cdots, n\}$.

Theorem 4 (Optimal Pricing Schedule)**.** Given $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\omega}}$, the following is an optimal pricing schedule for the platform:

(i) $T_j^F(\boldsymbol{m}) = T_i^W(\boldsymbol{m}) = 1 - (\hat{\omega}_i - \hat{\theta}_j)^2$ if $rank(\hat{\theta}_j | \hat{\boldsymbol{\theta}}) = rank(\hat{\omega}_j | \hat{\boldsymbol{\omega}})$ and

(ii) $T_j^F(\boldsymbol{m}) = T_i^W(\boldsymbol{m}) = 1$ otherwise,

where $rank(\theta_j | \boldsymbol{\theta})$ denotes the rank of θ_j in $\boldsymbol{\theta}$.

Proof. See **Appendix** for the proof. ■

It is worth mentioning that the optimal pricing schedule support an unraveling equilibrium and, consequently, extracts full surplus of the users. As shown in the example, the key element that makes the unraveling of locations possible is the increasing difference of Q^G, the expected utility of a user in group G when she successfully pretends to be in another location. When Q^G exhibits increasing difference, $wcl(m^G|Q^G)$ is non-empty for any message m^G. Thus, the platform can rationally infer that a user who has sent a message, m^G, would have been located at the worst-case location possible, and by definition of the worst-case location, users cannot be better off by pretending to be in another location.

6. Extension: Two-way Cheap-talk Communication Protocol

Matching platforms that operate in some areas, such as online dating markets, do not require users to provide a verifiable information. Upon registering for the service, the users upload their information, which is usually non-verifiable, to the platform through a private channel, and the platform suggests a recommended partner to the user. I model this communication pattern by a two-way cheap-talk communication protocol.

6.1. Non-Trivial Coarse Matching: Existence Theorem

Under the two-way cheap-talk communication protocol, messages from both users and the platform are costless and non-verifiable.

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1. Each user sends a message to the platform.
2. After receiving messages from agents, the platform generates a recommendation for workers.
3. After communication between the users and the platform, workers apply for jobs, and firms decides either accept or reject application.

Table 2: Cheap-talk communication protocol

Using the communication protocol, I provide an existence result of coarse matching of McAfee (2002) in games with a finite number of players, and provide an implementation methods of the coarse matching.

Definition 4 (Non-trivial coarse matching)**.** Given two equal-sized populations, a matchmaker sorts the populations into a finite number of classes before randomly matching the users within the classes. Coarse matching is said to be non-trivial if the number of classes is greater than one.

Note that in this communication game with cheap talk, I assume that recommendation is given only to one side of the market.

Proposition 2. A non-trivial coarse matching which strictly improves the platform's ex-post profit exists for any n, F and G.

Proof. See **Appendix** for the proof. ■

In the non-trivial coarse matching, platform categorize users on each side into two groups, recommends a matching partner within the same group on the other side to achieve a coarse matching and prevent unnecessary congestions. While most studies in coarse matching assumes a centralized matching procedure, the above application shows that an introduction of a platform enables us to achieve a coarse matching with a cheap-talk communication.

7. Conclusion

I studied a class of communication games in two-sided matching with an intermediary. Within the platform, instead of being matched with another user, users have the freedom to choose their match partner; however, they have limited knowledge about the other users. I have demonstrated that a platform, as an information gatekeeper, can elicit full information from users via verifiable messages, and subsequently persuade them via cheap talk to choose a partner of the platform's choosing. This, in turn, achieves a socially optimal matching outcome under certain conditions. The analysis delivers a sufficient condition and a necessary condition in terms of the shape of private information and number of users. I find that the key factors by which it is possible to achieve a profit maximizing matching outcome are a pool that consists of a large number of users, the proximity of distributions from which the users' tastes are drawn, and the uncertainty measure of the distributions.

The above analysis is worth extending in a few research directions. One direction is to focus on the two-way cheap-talk communication model between users and a platform. Although I have delivered a positive result in terms of implementing a coarse matching, I do not know yet how much of detailed information can be transmitted in other equilibria with finer partitions. Another direction is to consider a platform that operates according to different objectives or requires users to make payment to access certain information.

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Appendix

1. Proof of Remark

Remark. An optimal matching is achieved when a worker with $\hat{\omega}_{(i;n)}$ is matched with a firm with $\hat{\theta}_{(i;n)}$, $\forall i \in \{1, \cdots, n\}$.

Proof. I prove it by induction on n.
Base step: $n = 2$

$$\begin{aligned}(\hat{\theta}_{(1;2)} - \hat{\omega}_{(1;2)})^2 + (\hat{\theta}_{(2;2)} - \hat{\omega}_{(2;2)})^2 =&(\hat{\theta}_{(1;2)} - \hat{\omega}_{(2;2)} + \hat{\omega}_{(2;2)} - \hat{\omega}_{(1;2)})^2 \\ &+ (\hat{\theta}_{(2;2)} - \hat{\omega}_{(1;2)} + t_{(1;2)} - \hat{\omega}_{(2;2)})^2 \\ =&(\hat{\theta}_{(1;2)} - \hat{\omega}_{(2;2)})^2 + (\hat{\theta}_{(2;2)} - \hat{\omega}_{(1;2)})^2 \\ &- 2(t_{(2;2)} - \hat{\omega}_{(1;2)})(\hat{\theta}_{(2;2)} - \hat{\theta}_{(1;2)}) \\ \leq&(\hat{\theta}_{(1;2)} - \hat{\omega}_{(2;2)})^2 + (\hat{\theta}_{(2;2)} - \hat{\omega}_{(1;2)})^2\end{aligned}$$

Induction step: Suppose that $(\hat{\theta}_{(1;n)}, \cdots, \hat{\theta}_{(n;n)})$ and $(\hat{\omega}_{(1;n)}, \cdots, \hat{\omega}_{(n;n)})$ are given. Match $\hat{\theta}_{(1;n)}$ with $\hat{\omega}_{(k;n)}$ for some $k \in \{1, \cdots, n\}$. The rest $(\hat{\theta}_{(2;n)}, \cdots, \hat{\theta}_{(n;n)})$ and $(\hat{\omega}_{(1;n)}, \cdots, t_{(k-1;n)}, \hat{\omega}_{(k+1;n)} \cdots, t_{(n;n)})$ can be matched optimally when $\hat{\theta}_{(j;n)}$ is matched with $\hat{\omega}_{(j-1;n)}$ if $j \leq k$ and $\hat{\theta}_{(j;n)}$ is matched with $\hat{\omega}_{(j;n)}$ if $j > k$ by induction hypothesis. Now I will show $k = 1$ minimizes the distance sum. To show this, it is sufficient to show that $k \neq n$. It is because if $k \neq n$, I can apply the induction hypothesis to $(\hat{\theta}_{(1;k)}, \cdots, \hat{\theta}_{(k;k)})$ and $(\hat{\omega}_{(1;k)}, \cdots, \hat{\omega}_{(k;k)})$, which yields $k = 1$. Suppose $k = n$ and note that

$$(\hat{\theta}_{(1;n)} - \hat{\omega}_{(n;n)})^2 + (\hat{\theta}_{(n;n)} - \hat{\omega}_{(n-1;n)})^2 \geq (\hat{\theta}_{(1;n)} - \hat{\omega}_{(n-1;n)} + \hat{\omega}_{(n;n)} - \hat{\omega}_{(n;n)})^2.$$

Thus, I have

$$\begin{aligned}&(\hat{\theta}_{(2;n)} - \hat{\omega}_{(1;n)})^2 + \cdots + (\hat{\theta}_{(n;n)} - \hat{\omega}_{(n-1;n)})^2 + (\hat{\theta}_{(1;n)} - \hat{\omega}_{(n;n)})^2 \\ \geq&(\hat{\theta}_{(2;n)} - \hat{\omega}_{(1;n)})^2 + \cdots + (\hat{\theta}_{(n;n)} - \hat{\omega}_{(n;n)})^2 + (\hat{\theta}_{(1;n)} - \hat{\omega}_{(n-1;n)})^2,\end{aligned}$$

which contradicts the induction hypothesis. ■

2. Proof of Theorem 1

Theorem 1. $\exists N$ such that $\forall n \geq N$, an unraveling equilibrium exists in Γ^n if $\mathbf{Kolm}(F^{-1}, G^{-1}) < \min\{\sigma(F), \sigma(G)\}$.

Proof. Suppose that the sufficient conditions are satisfied. To find an *unraveling* equilibrium, I work backward from the users' partner choice problems. Consider firm j who is recommended to accept worker i. Firm j's belief on the location of worker i after communication can be denoted by a random variable, $W_n^i(\theta_j)$, which condition on firm j's reported location θ_j. Given all users' full revelation to the platform and platform's assortative recommendation, the recommendation only reveals that firm j and worker i have the same rank[14] in $\boldsymbol{\theta} = (\theta_1, \theta_2, \cdots, \theta_n)$ and $\boldsymbol{\omega} = (\omega_1, \omega_2, \cdots, \omega_n)$, respectively. Since the probability that θ_j is rank k is given by $F_{(k-1;n-1)}(\theta_j) - F_{(k;n-1)}(\theta_j)$.[15], the associated pdf of $W_n^i(\theta_j)$ is the following:

$$\sum_{k=1}^{n} [F_{(k-1;n-1)}(\theta_j) - F_{(k;n-1)}(\theta_j)] g_{(k;n)}(\omega).$$

which is equivalent to

$$\sum_{k=1}^{n} \binom{n-1}{k-1} F(\theta_j)^{k-1}(1 - F(\theta_j))^{n-k} g_{(k;n)}(\omega).$$

On the other hand, the location of worker $l \neq i$ can be denoted by a random variable $W_n^{-i}(\theta_j)$, and its associated pdf is

$$\sum_{k=1}^{n} \binom{n-1}{k-1} F(\theta_j)^{k-1}(1 - F(\theta_j))^{n-k} \frac{1}{n-1}\Big[ng(\omega) - g_{(k;n)}(\omega)\Big].$$

It is because 1) the pdf of the location of a worker whose rank is not k is $\frac{1}{n-1}\sum_{l \neq k} g_{(l;n)}(\omega)$. 2) the mean of the order statistic density $g_{(k;n)}$ is g. Therefore, I should have

$$ng(\omega) = \sum_{l=1}^{n} g_{l;n}(\omega) = (n-1)\Bigg[\frac{1}{n-1}\sum_{l \neq k} g_{(l;n)}(\omega)\Bigg] + g_{(k;n)}(\omega),$$

which enables us to express the location of $l \neq i$ in a closed form. Now, if multiple workers including i apply to firm j, j accept i *if and only if*

$$\mathbb{E}[1 - (\theta_j - W_n^i(\theta_j))^2] \geq \mathbb{E}[1 - (\theta_j - W_n^{-i}(\theta_j))^2].$$

Using the closed form for the location of worker $l \neq i$, the condition is equivalent to the following inequality:

$$\mathbb{E}[1 - (\theta_j - W_n^i(\theta_j))^2] \geq \mathbb{E}_\omega[1 - (\theta_j - \omega))^2].$$

Similarly, consider worker i who is recommended to apply to firm j. Denote the location of firm j by $\Theta_n^j(\omega_i)$. The worker has an incentive to follow the

[14] θ_j has rank k in $\boldsymbol{\theta}$ if it is the k^{th} smallest value in $\boldsymbol{\theta}$

[15] The probability that $k-1$ firms among $n-1$ firms are lower than θ_j, subtracting the probability that k firms among $n-1$ firms are lower than θ_j.

recommendation if

$$\mathbb{E}[1-(\Theta_n^j(\omega_i)-\omega_i)^2] \geq \mathbb{E}_\theta[1-(\theta-\omega_i))^2].$$

Lemma 1. $W_n^i(\theta) \xrightarrow{d} G^{-1}(F(\theta)), \forall\theta \in [0,1]$ and $\Theta_n^j(\omega) \xrightarrow{d} F^{-1}(G(\omega)), \forall\omega \in [0,1]$.

Proof. I shall show that $W_n^i(\theta) \xrightarrow{d} G^{-1}(F(\theta))$. The same proof can be applied to the second argument. First, note that if $\sqrt{n}(\frac{k}{n}-q) \to 0$, then $\omega_{(k;n)} \xrightarrow{d} G^{-1}(q)$ for $q \in (0,1)$.[16] Now, for each θ, define a random variable Q_θ^n whose value is k/n with probability $\binom{n-1}{k-1}F(\theta)^{k-1}(1-F(\theta))^{n-k}$ for $k = 1, \cdots, n$. I will show that

$$Q_\theta^n \xrightarrow{p} F(\theta).$$

The mean of Q_θ^n is

$$\mathbb{E}[Q_\theta^n] = \sum_{i=1}^{n} \frac{i}{n}\binom{n-1}{i-1}F(\theta)^{k-1}(1-F(\theta))^{n-k} = \frac{n-1}{n}F(\theta)$$

The second equality derived using a property of the mean of a binomial distribution.[17] Using a property of the variance of a binomial distribution, I find

$$\text{Var}[Q_\theta^n] = \frac{n-1}{n^2}F(\theta)(1-F(\theta)).$$

Thus,

$$\begin{aligned}\mathbb{P}[|Q_\theta^n - \mathbb{E}[Q_\theta^n]|] =& \mathbb{E}[\mathbf{1}_{\frac{(Q_\theta^n-\mathbb{E}[Q_\theta^n])^2}{\epsilon^2}\leq 1}] \\ \leq& \frac{1}{\epsilon^2}\mathbb{E}[(Q_\theta^n - \mathbb{E}[Q_\theta^n])^2] = \frac{1}{\epsilon^2}\text{Var}[Q_\theta^n]\end{aligned}$$

Since the last term tends to zero as $n \to \infty$, I have $Q_\theta^n \xrightarrow{p} F(\theta)$. Combining the two convergence results, the pdf of $W_n^i(\theta_j)$ converges to a function h, where $h(\theta) = 0$ if $\theta \neq \theta_j$, where θ_j is the only point of discontinuity. Thus, I have $W_n^i(\theta) \xrightarrow{d} G^{-1}(F(\theta))$. ■

Since the utility function is continuous and bounded, I can appeal to **Portmanteu theorem**. Therefore, the utility of the firm j from following recommendation converges to $1-(\theta_j - G^{-1}(F(\theta_j)))^2$.Now, I will use **Lemma 2** to show that this convergence of expected utilities are uniformly convergent.

[16] For a proof, see p.93 of DasGupta (2008).
[17] Mean of $B(m,p)$ is mp.

Lemma 2 (Test for uniform convergence)**.** Given a $\sum_{k=1}^{n} a_k(x)b_k(x)$, if $\{a_k(x)\}$ is uniformly bounded and is monotone for each x, while the series $\sum b_k(x)$ is uniformly convergent, then $\sum a_k(x)b_k(x)$ is also uniformly convergent.

Proof. See Hardy (1918) ■

For $\theta \in [0,1]$, define $a_k(\theta_j) = \int_0^1 \omega^m g_{(k;n)}(\omega)d\omega$, which is independent of θ, and $b_k(\theta) = \binom{n-1}{k-1}F(\theta)^{k-1}(1-F(\theta))^{n-k}$. Then, I have $\sum_{k=1}^{n} b_k(\theta) = 1$, $\forall n$ and $\forall \theta$, by the binomial theorem. On the other hand, $a_k(\theta)$ is monotonically increases in k and bounded below by 0 and above by 1 for any $m \in \{0,1,2,\cdots\}$. Since sum of two uniformly convergent functions also uniformly convergent, I conclude that the firm's expected utility converges uniformly to $1-(\theta_j - G^{-1}(F(\theta_j)))^2$ if it follows the recommendation.

Now, by the sufficient condition, I have

$$(F^{-1}(x) - G^{-1}(x))^2 < \sigma(F)^2, \ \forall x \in [0,1]$$

Since F has a unit support, the above inequality still holds when $x = F(\theta_j)$, $\forall \theta_j \in [0,1]$. Thus, the above inequality is equivalent to

$$(\theta_j - G^{-1}(F(\theta_j)))^2 < \sigma(F)^2, \ \forall \theta_j \in [0,1]. \tag{2}$$

On the other hand, since the variance minimizes the distance, I have

$$\sigma(F)^2 = \mathbf{Var}(F) \leq \mathbb{E}_\omega[(\theta_j - \omega)^2], \ \forall \theta_j \in [0,1]. \tag{3}$$

Combining the two inequalities above, I conclude that

$$1 - (\theta_j - G^{-1}(F(\theta_j)))^2 > 1 - \mathbb{E}_\omega[(\theta_j - \omega)^2].$$

Define $\epsilon_F = \sigma(F) - \mathbf{Kolm}(F^{-1}, G^{-1})$. By the uniform convergence result above, for ϵ_F, there exists an $N_F \in \mathbb{N}$ such that if the population of users on each side is larger than N_F, they do not have an incentive to deviate from accepting the recommended worker. Similarly, for $\epsilon_G = \sigma(G) - \mathbf{Kolm}(F^{-1}, G^{-1})$, there exists N_G such that all workers follow recommendation if the size of population is large than N_G. Since any effective deviation should be bilateral, I can consider the case at least one side of users do not have incentive deviate from following the platform's recommendation.

Now, consider the platform's strategy. As noted at Remark, a positive assortative matching maximizes the social surplus and the platform's utility. Thus, given that all users fully reveal their information and follow the recommendations, providing an assortative recommendation is optimal to the platform.

Finally, I need to construct a belief system which support full revelation of information. Suppose that a firm with θ who successfully pretends to have a location of θ' while other users fully reveal their locations. Its expected utility from the successful deception is $v(\theta,\theta') = 1 - \mathbb{E}[(\theta - W_n(\theta'))^2]$. Denote the derivative of v with respect to the second argument by v_2. I start by partitioning the unit interval as follows:

$$I_{[0,1]} = \cup_{l=1,2,\cdots} P_l$$

where 1) each P_l is an interval, 2) if $\theta_1, \theta_2 \in P_l$, then $v_2(\theta_1,\theta_1) \cdot v_2(\theta_2,\theta_2) \geq 0$, $\forall l \in \{1, 2, \cdots\}$ and 3) if $\theta_1 \in P_l$ and $\theta_2 \in P_{l+1}$, then $v_2(\theta_1,\theta_1) \cdot v_2(\theta_2,\theta_2) \leq 0$.[18] The unit interval is partitioned based on the deception incentives of users. If two locations are in the same interval, they locally want to be believed as locations in the the same direction. Now, suppose that a message $m \subset [0,1]$ is received from firm j.

I find the *locally* worst case location from each interval, and then show that there always exists at least one *globally* worst case location among them. For each $l \in \{1, 2, \cdots\}$, if $m \cap P_l \neq \emptyset$, I pick one type from $m \cap P_l$ by

$$\tilde{\theta}_l = \begin{cases} \min m \cap P_l & \text{if} \quad v_2(x,x) \geq 0, \forall x \in P_l \\ \max m \cap P_l & \text{if} \quad v_2(x,x) \leq 0, \forall x \in P_l. \end{cases}$$

Define $\tilde{\Theta}$ be the collection of such $\tilde{\theta}_l$. The platform's belief assigns probability 1 to a location $\tilde{\theta} \in \tilde{\Theta}$ which satisfies

$$\nexists \tilde{\theta}' \in \tilde{\Theta} \ s.t. \ v(\tilde{\theta}', \tilde{\theta}) > v(\tilde{\theta}', \tilde{\theta}').$$

That is, no other location in $\tilde{\Theta}$ wants to masquerade as $\tilde{\theta}$ even if it is possible to do so.[19] If there are multiple types that satisfies the above condition, I can pick one arbitrarily.

Before I show existence of such $\bar{\theta}$, I show that a firm with $\tilde{\theta}_l \in P_l$ cannot benefit from pretending to be another location which is available to him, I first consider a local incentive within P_l and then global incentive within $\tilde{\Theta}$. Sup-

[18] The partition is well defined because by continuity of $v_2(\cdot,\cdot)$.

[19] Although the construction of belief is motivated from Hagenbach et al. (2014), the construction cannot directly applied to the model because I deal with continuum type space with non-monotonic masquerade relation.

pose first that $v_2(x,x) \geq 0,\ \forall x \in P_l$.[20] Since $(k-1)\binom{n-1}{k-1} = (n-1)\binom{n-2}{k-2}$ and $(n-k)\binom{n-1}{k-1} = (n-1)\binom{n-2}{k-1}$, I have

$$\begin{aligned}
&\frac{\partial}{\partial\theta'}\binom{n-1}{k-1}F(\theta')^{k-1}(1-F(\theta'))^{n-k}\\
=&\binom{n-1}{k-1}\Big[(k-1)F(\theta')^{k-2}(1-F(\theta'))^{n-k} - (n-k)F(\theta')^{k-1}(1-F(\theta'))^{n-k-1}\Big]f(\theta')\\
=&(n-1)\Big[\binom{n-2}{k-2}F(\theta')^{k-2}(1-F(\theta'))^{n-k} - \binom{n-2}{k-1}F(\theta')^{k-1}(1-F(\theta'))^{n-k-1}\Big]f(\theta').
\end{aligned}$$

Thus, v_2 can be reformulated as

$$\begin{aligned}
v_2(\theta,\theta') =&\frac{\partial}{\partial\theta'}\sum_{k=1}^{n}\binom{n-1}{k-1}F(\theta')^{k-1}(1-F(\theta'))^{n-k}\mathbb{E}[1-(\theta-\omega_{(k;n)})^2]\\
=&\sum_{k=2}^{n}(n-1)f(\theta')\binom{n-2}{k-2}F(\theta')^{k-2}(1-F(\theta'))^{n-k}\mathbb{E}[1-(\theta-\omega_{(k;n)})^2]\\
&-\sum_{k=1}^{n-1}(n-1)f(\theta')\binom{n-2}{k-1}F(\theta')^{k-1}(1-F(\theta'))^{n-k-1}\mathbb{E}[1-(\theta-\omega_{(k;n)})^2]\\
=&(n-1)f(\theta')\sum_{k=1}^{n-1}\binom{n-2}{k-1}F(\theta')^{k-1}(1-F(\theta'))^{n-k-1}\mathbb{E}[(\theta-\omega_{(k;n)})^2-(\theta-\omega_{(k+1;n)})^2]\\
=&(n-1)f(\theta')\sum_{k=1}^{n-1}\binom{n-2}{k-1}F(\theta')^{k-1}(1-F(\theta'))^{n-k-1}\int_0^1(\theta-\omega)^2[g_{(k;n)}(\omega)-g_{(k+1;n)}(\omega)]d\omega\\
=&(n-1)f(\theta')\sum_{k=1}^{n-1}\binom{n-2}{k-1}F(\theta')^{k-1}(1-F(\theta'))^{n-k-1}[\alpha_k\theta+\beta_k].
\end{aligned}$$

where $\alpha_k = -2\int_0^1\omega[g_{(k;n)}(\omega)-g_{(k+1;n)}(\omega)]d\omega$ and $\beta_k = \int_0^1\omega^2[g_{(k;n)}(\omega)-g_{(k+1;n)}(\omega)]d\omega$. I have $\alpha_k > 0$ and $\beta_k < 0$ because $G_{(k+1;n)}$ first order stochastically dominates $G_{(k;n)}$. Thus, if $\theta_1,\theta_2 \in P_l$ and $\theta_1 < \theta_2$ then

$$0 \leq v_2(\theta_1,\theta_1) \leq v_2(\theta_1,\theta') < v_2(\theta_2,\theta'),\ \forall\theta' \in [\theta_1,\theta_2],$$

and, as a consequence,

$$v(\theta_2,\theta_1) = \int_0^{\theta_1} v_2(\theta_1,\theta)d\theta < \int_0^{\theta_2} v_2(\theta_1,\theta)d\theta = v(\theta_2,\theta_2)$$

As $\tilde{\theta}_l \in P_l$ and $v_2(x,x) \geq 0,\ \forall x \in P_l$, the firm only can pretend to have a location θ' which is lower than $\tilde{\theta}_l$ if it pretends to have another location in P_l. However, the inequality above shows that the deception within P_l cannot be beneficial.

[20] The same proof applies to the other case.

Now, consider the mimicking incentive within $\tilde{\Theta}$. The only case when $\tilde{\theta}_l$ firm can profitably pretend to have another location is that when there is a cycle of the form:

$$v(\tilde{\theta}_l,\tilde{\theta}_l) < v(\tilde{\theta}_l,\tilde{\theta}_{m_1}),\ v(\tilde{\theta}_{m_1},\tilde{\theta}_{m_1}) < v(\tilde{\theta}_{m_1},\tilde{\theta}_{m_2}),\ \cdots,\ \ v(\tilde{\theta}_{m_t},\tilde{\theta}_{m_t}) < v(\tilde{\theta}_{m_t},\tilde{\theta}_l).$$

I will show that there cannot be a cycle of any length.

Lemma 3. If $\exists(m_1, m_2, \cdots, m_t)$ s.t. $v(\tilde{\theta}_l,\tilde{\theta}_l) < v(\tilde{\theta}_l,\tilde{\theta}_{m_1})$ and $v(\tilde{\theta}_{m_k},\tilde{\theta}_{m_k}) < v(\tilde{\theta}_{m_k},\tilde{\theta}_{m_k+1})$, for $k = 1, 2, \cdots, t-1$, then $v(\tilde{\theta}_{m_t},\tilde{\theta}_{m_t}) \geq v(\tilde{\theta}_{m_t},\tilde{\theta}_l)$.

Proof. First, notice that $v(\theta,\theta')$ exhibits an increasing difference.

$$\begin{aligned}\frac{\partial^2}{\partial\theta\partial\theta'}v(\theta,\theta') =& \frac{\partial}{\partial\theta}(n-1)f(\theta')\sum_{k=1}^{n-1}\binom{n-2}{k-1}F(\theta')^{k-1}(1-F(\theta'))^{n-k-1}[\alpha_k\theta+\beta_k]\\ =&(n-1)f(\theta')\sum_{k=1}^{n-1}\binom{n-2}{k-1}F(\theta')^{k-1}(1-F(\theta'))^{n-k-1}\alpha_k,\end{aligned}$$

where $\alpha_k = 2\int_0^1 \omega[g_{(k+1;n)}(\omega) - g_{(k;n)}(\omega)]d\omega > 0$. Now, I will prove the statement by induction on the length of a cycle.

Base step: No cycle of length 2.
Let $\theta_1 < \theta_2$. By increasing difference, $v(\theta_2,\theta_1) - v(\theta_1,\theta_1)$ is non-decreasing in the second argument:

$$v(\theta_1,\theta_2) - v(\theta_1,\theta_1) \leq v(\theta_2,\theta_2) - v(\theta_2,\theta_1).$$

Thus, if $v(\theta_1,\theta_2) \leq v(\theta_1,\theta_1)$, then $v(\theta_2,\theta_2) \leq v(\theta_2,\theta_1)$. That is, if θ_1 benefits from mimicking θ_2, then θ_2 cannot benefit from mimicking θ_1.

Induction step: Suppose that there is no cycle of length $l-1$
For a contrary, suppose that there is a cycle of length l with $\theta_1, \theta_2, \cdots, \theta_l$. Without loss of generality, let $\theta_1 = \min\{\theta_1, \cdots, \theta_l\}$ and cycle connects θ_i to θ_{i+1} and θ_l to θ_1, for all $i = 1, \cdots, l-1$. By increasing difference,

$$v(\theta_1,\theta_2) - v(\theta_1,\theta_1) \leq v(\theta_k,\theta_2) - v(\theta_k,\theta_2),\ \forall k = 1, \cdots, l.$$

Thus, I have $v(\theta_k,\theta_2) \geq v(\theta_k,\theta_1) > v(\theta_k,\theta_k)$ which implies there is a cycle of length $l-1$. ■

Since there is no cycle, any $\tilde{\Theta}$ has an element $\tilde{\theta}$ s.t. $\nexists\tilde{\theta}' \in \tilde{\Theta}$ *s.t.* $v(\tilde{\theta}',\tilde{\theta}) > v(\tilde{\theta}',\tilde{\theta}')$, which completes the proof. ■

Proof of Theorem 2

Theorem 2. If there exists N s.t. an unraveling equilibrium exist in Γ^n, $\forall n \geq N$, then $\nexists x^* \in [0,1]$ s.t.

$$p > \frac{1-(F^{-1}(x^*)-G^{-1}(x^*))^2}{1-\mathbb{E}_y[(F^{-1}(y)-G^{-1}(x^*))^2]}.$$

Proof. Suppose for a contrary that $\exists x^* \in [0,1]$ that satisfies the inequality. The expected utility of a worker with $\omega^* = G^{-1}(x^*)$ from following the recommendation converges to $1-(F^{-1}(x^*)-G^{-1}(x^*))^2$. If he apply to a firm which is not recommended to him, his expected utility converges to $p\big[1-\mathbb{E}_\theta[(\omega^*-\theta)^2]\big]$. Thus, the worker benefits from deviation and applying to another firm. ■

Proof of Proposition 1

Proposition 1. An unraveling equilibrium exist in Γ^∞ *if and only if* $\nexists x^* \in [0,1]$ s.t. $p > \frac{1-(F^{-1}(x^*)-G^{-1}(x^*))^2}{1-\mathbb{E}_y[(F^{-1}(y)-G^{-1}(x^*))^2]}$.

Proof. (Sufficiency) I work backward to verify that an unraveling equilibrium indeed exists under the condition. Consider a worker a who is recommended to apply to firm b. By assortative recommendation and truthful revelation, the worker knows that the location of the firm is $F^{-1}(G(\omega_a))$. His updated belief on the location of the other firms remains the same because the revelation of a location of one firm does not reveal any information about the locations of other firms when there are continuum firms. Since $p[1-\mathbb{E}_\theta[(\omega_a-\theta)^2]] \leq 1-(\omega_a - F^{-1}(G(\omega_a)))^2$, the worker does not have incentive to deviate. Since all firms receive only one application, they will accept their applicant.

Now, to apply "skepticism", I need to construct a belief system of the platform which makes users have no incentive to withhold any information. Denote the set of roots of $(F-G)(x)$ by[21] R and suppose that a message m is received from a firm[22]. Given message m from a user, consider a collection of subsets of m, $\{m_\alpha\}_{\alpha\in\mathscr{I}_p}$, which is constructed in a way that

$$\text{if } \theta < \theta' \text{ and } \theta, \theta' \in m_{\alpha'} \text{ for some } \alpha' \in \mathscr{I}_p, \text{ then } \nexists r \in R \; s.t. \; \theta \leq r \leq \theta'.$$

Thus, $\{m_\alpha\}_{\alpha\in\mathscr{I}_p} \cup (R\cap m)$ constitutes a partition of m. By construction, if $\theta \in m_{\alpha'}$ for some $\alpha' \in \mathscr{I}_p$, then it is either $F(\theta) > G(\theta)$ or $F(\theta) < G(\theta)$. Define $l_{\alpha'}$ by

[21] R is a totally ordered set as it is a subset of $[0,1]$

[22] The same construction works for workers as well.

$\sup m_{\alpha'}$ if $F(\theta) > G(\theta)$ and $\inf m_{\alpha'}$ if $F(\theta) < G(\theta)$. Let $\{l_\alpha\}_{\alpha \in \mathscr{I}_p} \cup (R \cap m) = l(m)$. The platform believes the user's location to be

$$\arg \min_{x \in l(m)} \left\{ -\left(x - G^{-1}(F(x)) \right)^2 \right\} \tag{4}$$

If there are multiple arguments which solves (4), pick one arbitrarily. I need to show that truthful revelation constitutes an equilibrium. Suppose that user with a location θ tries to mimic $\theta' \neq \theta$. First of all, it should be

$$|\theta' - G^{-1}(F(\theta'))| > |\theta - G^{-1}(F(\theta))| \tag{5}$$

Otherwise, θ' wouldn't be taken over θ by the platform. Furthermore, it should be

$$|\theta - G^{-1}(F(\theta'))| = |\theta - \theta'| + |\theta' - G^{-1}(F(\theta'))| > |\theta - \theta'| + |\theta - G^{-1}(F(\theta))|.$$

The first equality is from the construction of belief system and the second inequality is by inequality (5). Thus, firms do not have incentive to withhold any information, and this completes the proof.

(Necessity) Suppose for a contrary that $\exists x^*$ s.t. $p[1 - \mathbb{E}_y[(F^{-1}(y) - G^{-1}(x^*))^2]] > 1 - (F^{-1}(x^*) - G^{-1}(x^*))^2$. Then, a worker with a location $\omega = G^{-1}(x^*)$ benefits from deviating and applying to a firm which is not recommended to him. Thus, an unraveling equilibrium cannot exist. ■

Theorem 3. Stable matching is attainable if

$$\max\{f(x), g(x)\} \leq 2 \min\{f(x), g(x)\},\ \forall x \in [0, 1].$$

Proof. Suppose the distributions G and F satisfy Assumption 2, Assumption 3 and the sufficient condition presented in the Theorem. Define $R^G(x, x')$ to be the expected utility of a user of location x in group G when she successfully convinces other users that her location is x' while other users truthfully and fully reveal their locations. I first show that $R^W(\omega, \omega')$ exhibits increasing difference. One can show the increasing difference of $R^F(\theta, \theta')$ using the same logic. In a fair stable matching, $R^W(\omega, \omega')$ is

$$1 - \frac{\min\{f(\omega'), g(\omega')\}}{\max\{f(\omega'), g(\omega')\}}(\omega - \omega')^2 - \left(1 - \frac{\min\{f(\omega'), g(\omega')\}}{\max\{f(\omega'), g(\omega')\}}\right)(\omega - F^{-1}(1 - G(\omega')))^2.$$

■

Because of (MLRP) and (Mirror), $f(x) \geq g(x)$ if and only if $x \leq \frac{1}{2}$. Thus,

$R^W_{12}(\omega, \omega')$ is

$$\begin{cases} \frac{d}{d\omega'}\left(\frac{g(\omega')}{f(\omega')}\right)(\omega' - F^{-1}(1-G(\omega'))) + \frac{g(\omega')}{f(\omega')} - \frac{g(\omega')(f(\omega')-g(\omega'))}{f(\omega')f(F^{-1}(1-G(\omega')))} & \text{if } \omega' \leq \frac{1}{2} \\ \frac{d}{d\omega'}\left(\frac{f(\omega')}{g(\omega')}\right)(\omega' - F^{-1}(1-G(\omega'))) + \frac{f(\omega')}{g(\omega')} - \frac{g(\omega')-f(\omega')}{f(F^{-1}(1-G(\omega')))} & \text{otherwise.} \end{cases}$$

By (Mirror), $1 - G(\omega') = F(1 - \omega')$. Thus, the first terms in the above two expressions are positive as $\frac{d}{d\omega'}\left(\frac{g(\omega')}{f(\omega')}\right)$ is increasing and $\frac{d}{d\omega'}\left(\frac{f(\omega')}{g(\omega')}\right)$ is decreasing. To demonstrate the last two terms are also positive, it is sufficient to show the following:

$$1 - \frac{f(\omega') - g(\omega')}{f(1-\omega')} \geq 0, \ \omega' \leq \frac{1}{2} \text{ and}$$
$$\frac{f(\omega')}{g(\omega')} - \frac{g(\omega') - f(\omega')}{f(1-\omega')} \geq 0, \ \omega' > \frac{1}{2}.$$

By (Mirror), they can further be simplified as

$$2g(\omega') - f(\omega') \geq 0, \ \omega' \leq \frac{1}{2} \text{ and}$$
$$2f(\omega') - g(\omega') \geq 0, \ \omega' > \frac{1}{2}.$$

It is clear to see the above inequalities are satisfied by the sufficient condition provided in the Theorem.

Now, since $R^W(\omega, \omega')$ exhibits increasing difference, each user can infer each other user's true location by applying skepticism as shown in the proof of Theorem 1, and thus, the game after communication is a complete information game which has a fair stable matching as an equilibrium outcome.

Theorem 4. Given $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\omega}}$, the following is an optimal pricing schedule to the platform:

(i) $T^F_j(\boldsymbol{m}) = T^W_i(\boldsymbol{m}) = 1 - (\hat{\omega}_i - \hat{\theta}_j)^2$ if $rank(\hat{\theta}_j|\hat{\boldsymbol{\theta}}) = rank(\hat{\omega}_j|\hat{\boldsymbol{\omega}})$ and

(ii) $T^F_j(\boldsymbol{m}) = T^W_i(\boldsymbol{m}) = 1$ otherwise.

Proof. Firstly, it is clear that a user who fully reveals her location and receives an optimal pricing schedule will apply to or accept a user with the same rank on the other side. To show it cannot benefit from pretending to have other locations, it is sufficient to show that Q^G_n exhibits increasing difference. If it does, the belief system that supports the equilibrium will follow as in the proof of Theorem 1. We show that Q_n^F exhibits increasing difference. Similar proof can be used to show increasing difference of Q_n^W.

$$\begin{aligned}\frac{d^2}{d\theta d\theta'}Q_n^F(\theta,\theta') =&\frac{d^2}{d\theta d\theta'}\mathbb{E}[(\theta' - W_n(\theta'))^2 - (\theta - W_n(\theta'))^2] \\ =&-\frac{d^2}{d\theta d\theta'}\mathbb{E}[(\theta - W_n(\theta'))^2] \\ =&v_{12}(\theta,\theta') > 0,\end{aligned}$$

where v is as defined in the proof of Theorem 1[23], and $v_{12} > 0$ is also shown in the proof of Theorem 1. ■

Proof of Proposition 2

Proposition 2. A non-trivial coarse matching exists for any n, F and G.

Proof. Consider an equilibrium with thresholds $\bar{\omega}$ and $\bar{\theta}$ in which all workers whose locations are lower than $\bar{\omega}$ send $\underline{m}$ and all workers with locations higher than $\bar{\omega}$ sends $\overline{m}$ to the platform. On the other hand, all firms with location lower than $\bar{\theta}$ send $\underline{s}$, while firms with location higher than $\bar{\theta}$ sends $\overline{s}$ to the platform. The platform, after collecting messages, recommends each user in an assortative manner. Let $\#m$ denote the number of message m collected by the platform: If $\#\bar{m} > \#\bar{s}$ and $\#\underline{m} \leq \#\underline{s}$, a worker who sent a lower message, $\underline{m}$, is recommended with a firm who sent a lower message, $\underline{s}$. If a worker sends a higher message, $\overline{m}$, then the platform arbitrarily picks $\#\overline{s}$ workers and each of them are recommended with a firm who sent a higher message, $\overline{s}$. All other workers are recommended with a firm who sent a lower message. If the other case, $\#\bar{m} \leq \#\bar{s}$ and $\#\underline{m} > \#\underline{s}$ happens, the platform provides recommendations in the opposite way. At the last stage of the game, all workers apply to its recommended firm, and all firms accept its applicant.

To show this indeed constitute an equilibrium, I will work backward from the individual's match choice problem. Suppose that a worker i sent $\underline{m}$, and recommended to apply to firm j. In equilibrium, the partial assortative recommendation reveals that $\theta_j \geq \bar{\theta}$ with probability

$$\sum_{k=0}^{n}\binom{n}{k}F(\bar{\theta})^k(1-F(\bar{\theta}))^{n-k}\sum_{j=k}^{n-1}\binom{n-1}{j}G(\bar{\omega})^j(1-G(\bar{\omega}))^{n-1-j}(1-\frac{k}{1+j}).$$

He also updates about the location of other firms, say firm $k \neq j$. By *ex-ante* homogeneity, the probability that j is recommended to i is $\frac{1}{n}$. Using the total law

[23] $v(\theta,\theta') = 1 - \mathbb{E}[(\theta - W_n(\theta'))^2]$

of expectation, $P[\theta_j \geq \bar{\theta}]$ is the same as

$$\frac{1}{n}\mathbb{P}[\theta_j \geq \bar{\theta}|j \text{ is recommended to } i] + \frac{n-1}{n}\mathbb{P}[\theta_j \geq \bar{\theta}|j \text{ is not recommended to } i]$$

By *ex-ante* homogeneity again,

$$P[\theta_j \geq \bar{\theta}|j \text{ is not recommended to } i] = P[\theta_k \geq \bar{\theta}|k \text{ is not recommended to } i]$$

Thus, $P[\theta_k \geq \bar{\theta}|k \text{ is not recommended to } i]$ is

$$\frac{n}{n-1}(1-F(\bar{\theta})) - \frac{1}{n-1}\mathbb{P}[\theta_j \geq \bar{\theta}|j \text{ is recommended to } i]$$

For notational simplicity, I denote $P[\theta_j \geq \bar{\theta}|j \text{ is recommended to } i]$ by $P[\theta_j \geq \bar{\theta}|j]$ and $P[\theta_k \geq \bar{\theta}|k \text{ is not recommended to } i]$ by $P[\theta_k \geq \bar{\theta}|\neg k]$. Given that $1 \geq \bar{1}$, applying to firm j is better than applying to other firm if

$$\begin{aligned} &\mathbb{P}[\theta_j \geq \bar{\theta}|j]\mathbb{E}[1-(\theta_j-\omega_i)^2|\theta_j \geq \bar{\theta}] + \mathbb{P}[\theta_j \leq \bar{\theta}|j]\mathbb{E}[1-(\theta_j-\omega_i)^2|\theta_j \leq \bar{\theta}] \\ \geq &\max\{0, \frac{1}{2}P[\theta_k \leq \bar{\theta}|\neg k]\mathbb{E}[1-(\theta_k-\omega_i)^2|\theta_k \leq \bar{\theta}] + P[\theta_k \geq \bar{\theta}|\neg k]\mathbb{E}[1-(\theta_k-\omega_i)^2|\theta_k \geq \bar{\theta}]\} \end{aligned}$$

Since $1 \geq \bar{1}$, I have

$$\mathbb{P}[\theta_j \geq \bar{\theta}|j]\mathbb{E}[1-(\theta_j-\omega_i)^2|\theta_j \geq \bar{\theta}] + \mathbb{P}[\theta_j \leq \bar{\theta}|j]\mathbb{E}[1-(\theta_j-\omega_i)^2|\theta_j \leq \bar{\theta}] \geq 0.$$

Using the closed form expression of $P[\theta_k \leq \bar{\theta}|\neg k]$ in terms of j, the remaining inequality can be simplified as follows:

$$[\mathbb{P}[\theta_j \leq \bar{\theta}|j]-F(\bar{\theta})]\mathbb{E}[1-(\theta_j-\omega_i)^2|\theta_j \leq \bar{\theta}] \geq [\mathbb{P}[\theta_j \leq \bar{\theta}|j]-F(\bar{\theta})]\mathbb{E}[1-(\theta_j-\omega_i)^2|\theta_j \geq \bar{\theta}]$$

Lemma 4. $\mathbb{P}[\theta_j \leq \bar{\theta}|j] > F(\bar{\theta}),\ \forall \bar{\theta} \in (0,1)$.

Proof. By a property of the binomial distribution, $B(n,p)$, it is

$$\sum_{k=0}^{n}\binom{n}{k}p^k(1-p)^{n-k}k = np.$$

On the other hand, using a binomial theorem, it is

$$\sum_{j=0}^{n-1}\binom{n-1}{j}G(\omega_i)^k(1-G(\omega_i))^{n-1-j} = 1.$$

Letting $p = F(\bar{\theta})$, and using the two equalities above, I can express $F(\bar{\theta})$ in a form of

$$F(\bar{\theta}) = \sum_{k=0}^{n} \binom{n}{k} F(\bar{\theta})^k (1 - F(\bar{\theta}))^{n-k} \frac{k}{n} \sum_{j=0}^{n-1} \binom{n-1}{j} G(\omega_i)^k (1 - G(\omega_i))^{n-1-j}.$$

Thus,

$$\begin{aligned}
&\mathbb{P}[\theta_j \leq \bar{\theta}|j] - F(\bar{\theta}) \\
=&\sum_{k=0}^{n} \binom{n}{k} F(\bar{\theta})^k (1 - F(\bar{\theta}))^{n-k} \sum_{j=0}^{k-1} \binom{n-1}{j} G(\omega_i)^k (1 - G(\omega_i))^{n-1-j} \Big(1 - \frac{k}{n}\Big) \\
+&\sum_{k=0}^{n} \binom{n}{k} F(\bar{\theta})^k (1 - F(\bar{\theta}))^{n-k} \sum_{j=k}^{n-1} \binom{n-1}{j} G(\omega_i)^k (1 - G(\omega_i))^{n-1-j} \Big(\frac{k}{1+j} - \frac{k}{n}\Big)\Big] > 0.
\end{aligned}$$

■

By **Lemma 4** I conclude that ω_i worker follows the recommendation if

$$\mathbb{E}[1 - (\theta_j - \omega_i)^2 | \theta_j \leq \bar{\theta}] \geq \mathbb{E}[1 - (\theta_j - \omega_i)^2 | \theta_j \geq \bar{\theta}].$$

Before I study the platform's incentive, I find that $\bar{\omega}$ and $\bar{\theta}$ should satisfy the *arbitrage condition* of Crawford and Sobel (1982). That is, by continuity and concavity, $\bar{\omega}$ is decided by the following condition:

$$\begin{aligned}
\mathbb{E}[1 - (\theta - \omega)^2 | \theta \leq \bar{\theta}] \geq& \mathbb{E}[1 - (\theta - \omega)^2 | \theta \geq \bar{\theta}] \ \forall \omega \leq \bar{\omega} \\
\mathbb{E}[1 - (\theta - \omega)^2 | \theta \leq \bar{\theta}] \leq& \mathbb{E}[1 - (\theta - \omega)^2 | \theta \geq \bar{\theta}] \ \forall \omega \geq \bar{\omega}.
\end{aligned}$$

Similarly, $\bar{\theta}$ is decided by the following condition:

$$\begin{aligned}
\mathbb{E}[1 - (\theta - \omega)^2 | \omega \leq \bar{\omega}] \geq& \mathbb{E}[1 - (\theta - \omega)^2 | \omega \geq \bar{\omega}] \ \forall \theta \leq \bar{\theta} \\
\mathbb{E}[1 - (\theta - \omega)^2 | \omega \leq \bar{\omega}] \leq& \mathbb{E}[1 - (\theta - \omega)^2 | \omega \geq \bar{\omega}] \ \forall \theta \geq \bar{\theta}.
\end{aligned}$$

Now, I need to check the incentive of the platform to provide partially assortative recommendation. Suppose that all users report their location using the binomial message space, and they all follow recommendations of the platform. Consider the situation where worker 1 sent $\bar{m}$, worker 2 sent $\underline{m}$, firm 1 sent $\bar{s}$, and firm 2 sent $\underline{s}$. Now, for a contrary, suppose that the platform recommends firm 1 to worker 2 and firm 2 to worker 1. Fixing the recommendations to the other workers, other than worker 1 and worker 2, the same, the platform's expected utility from this non-assortative matching is greater than an assortative matching by

$$\mathbb{E}[1-(\theta-\omega)^2|\omega \geq \bar{\omega} \wedge \theta \leq \bar{\theta}] + \mathbb{E}[1-(\theta-\omega)^2|\omega \leq \bar{\omega} \wedge \theta \geq \bar{\theta}]$$
$$-\mathbb{E}[1-(\theta-\omega)^2|\omega \leq \bar{\omega} \wedge \theta \leq \bar{\theta}] - \mathbb{E}[1-(\theta-\omega)^2|\omega \geq \bar{\omega} \wedge \theta \geq \bar{\theta}].$$

Here, by the *arbitrage conditions*, I have

$$-\mathbb{E}[1-(\theta-\omega)^2|\omega \leq \bar{\omega} \wedge \theta \leq \bar{\theta}] + \mathbb{E}[1-(\theta-\omega)^2|\omega \leq \bar{\omega} \wedge \theta \geq \bar{\theta}] < 0 \text{ and}$$
$$\mathbb{E}[1-(\theta-\omega)^2|\omega \geq \bar{\omega} \wedge \theta \leq \bar{\theta}] - \mathbb{E}[1-(\theta-\omega)^2|\omega \geq \bar{\omega} \wedge \theta \geq \bar{\theta}] < 0.$$

which makes the overall value negative. Thus, the platform cannot benefit from non-assortative recommendation. This complete the proof since the user's incentive not to deviate at reporting stage is satisfied by the *arbitrage condition.* ■